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On the Radiation of Gluon Jets: A Summary

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Abstract

Radiation of gluons gives rise to extra jets in top quark events that can lead to complications in event reconstruction and mass measurement. In this short paper we desire to do a basic introspection of the cancellation of infrared divergence via a pedagogical treatment. We first calculate the 1-loop vertex correction to *M* from virtual gluon. This computation is followed by the simplification of the three-body phase space integral. Thereafter, we compute the diff erential cross section from the process electron plus positron into quark-qurk-antigluon to lowest order in α and αg. Following this derivation, by assigning μ to be nonzero, we reevaluate the averaged squared amplitude. Last but not the least, we complete the integral computation over Feynman parameters and finally obtain our comprehensive result.

***Keywords*: Radiation, gluon jets, infrared divergence**.

Introduction

Quantum Chromodynamics (QCD) is a rich and fascinating theory. From a simple Lagrangian emerges numerous complex phenomena, such as confinement of quarks or gluons into hadrons, and jet production at high energies.

Quarks, gluons and anti-quarks are the constituents of protons, neutrons and by definition other hadrons. It is a fascinating aspect of the physics of our world that when one of these particles is kicked out of the hadron that contains it, flying out with high motion-energy, it is never observed macroscopically. Instead, a high-energy quark or anti-quark or gluon is transformed into a spray of hadrons. These are particles made from quarks, antiquarks and gluons. This spray is called a jet. We must note here that this statement applies to the five lighter flavors of quark, and not the top quark, which decays to a W particle and a bottom quark before a jet can form.

Hard jets provide powerful probes of the transient dense matter produced in central collisions of ultra-relativistic heavy nuclei. One of the most important discoveries in the relativistic heavy ion experiments at RHIC at Brookhaven National Laboratory and the LHC at CERN is the jet quenching phenomena [1-4], where high energy partons lose tremendous energy through their interactions with the quark-gluon plasma created in heavy ion collisions. Theoretically, the jet energy loss can be understood as a result of the induced gluon radiation when the parton traverses the hot QCD matter, and has been well formulated in the QCD frame-work [5-9].

Alternatively, the strong coupling feature of the medium can be described by models based on the Ads or CFT correspondence in string theory [10-15]. These calculations have been successfully applied to heavy ion phenomenology in order to understand the jet quenching related experimental data from the RHIC and LHC [16].

The goal of this paper is to perform a summarized and systematic study on the cancellation of infrared divergence. We perform this task with particular to the Radiation of Gluon Jets.

I. One-loop Vertex Correction

First we calculate the 1-loop vertex convection to *M* form gluon. The amplitude is given by

$$id_1M = Q_f(-ie)^2(ig)^2\bar{u}(p_1)\left[\int \frac{d^dk}{(2\pi)^d}\gamma^\nu \frac{i}{k}\gamma^\mu \frac{i}{k-q}\gamma_\mu \times \frac{-i}{(k-p_1)^2-\mu^2}\right]v(p_2)\frac{-i}{q^2}\bar{v}(k_2)\gamma^\mu u(k_1) \tag{1}$$

Now we simplify the loop integral in the standard way such that we have,

$$\begin{aligned} id_1M &= -ig^2\left[\int \frac{d^dk}{(2\pi)^d}\int_0^1 dx\int_0^{1-x} dy \frac{2\left(\frac{2-d}{d}(k')^2-2(1-x)(x+y)q^2\right)}{((k')^2-\Delta)^3}\right]\times iM_o \\ &= \frac{2g^2}{(4\pi)^{\frac{d}{2}}}\int_0^1 dx\int_0^{1-x} dy\left[\frac{(2-d)^2}{4\Delta^{2-d/2}}\Gamma(2-\frac{d}{2})+\frac{(1-x)(x+y)q^2}{\Delta^{3-d/2}}\times\Gamma(3-\frac{d}{2})\right] iM_o \\ &= \frac{2g^2}{(4\pi)^2}\int_0^1 dx\int_0^{1-x} dy\left[\frac{2}{\epsilon}-\gamma+\log 4\pi-\log\Delta-2+\frac{(1-x)(x+y)q^2}{\Delta}\right] iM_o \end{aligned} \quad (2)$$

where

$$iM_o = Q_f(-ie)^2\bar{u})p_1)\gamma^\mu\bar{v}(p_2)\frac{1}{q_2}v(k_2)\gamma_\mu u(k_1) \quad (3)$$

is the tree amplified, and

$$k' = k - xq - yp_1, \qquad \Delta = -x(1-x-y)q^2 - y(1-y)p_1^2 + y\mu^2 \quad (4)$$

with the external legs amputated, the result is,

$$i\delta_1 M = \frac{2g^2}{(4\pi)^2}\int_0^1 dx\int_0^{1-x} dy\left[\log\left(\frac{y\mu^2}{y\mu^2-x(1-x-y)q^2}\right)+\frac{(1-x)(x+y)q^2}{y\mu^2-x(1-x-y)q^2}\right] iM_o \quad (5)$$

Then, the cross section is given by

$$\sigma\left(e^+e^-\longrightarrow q\bar{q}+q\bar{q}g\right)=\frac{4\pi\alpha^2}{3s}\cdot 3\mid F_1(q^2=s)\mid^2 \quad (6)$$

with

$$\begin{aligned} F_1(q^2=s) &= Q_f^2+\frac{Q_f^2\alpha g}{2\pi}\frac{2g^2}{(4\pi)^2} \\ &\times\int_0^1 dx\int_0^{1-x} dy\left[\log\left(\frac{y\mu^2}{y\mu^2-x(1-x-y)s}\right)+\frac{(1-x)(x+y)s}{y\mu^2-x(1-x-y)s}\right] \end{aligned} \quad (7)$$

The Feynman integration will be carried out later in our computation.

II. Three-body Phase Space

Now, we simplify the 3-body phase space integral

$$\int d\Pi_3 = \int\frac{d^3k_1d^3k_2d^3k_3}{(2\pi)^9 2E_1 2E_2 2E_3}\delta^4(q-k_1-k_2-k_3) \quad (8)$$

in the center of mass com frame. It is convenient to introduce a new set of variables

$$x_i = 2k_i\cdot q/q^2, \qquad (i=1,2,3) \quad (9)$$

In the com frame,

$$x_i = 2E_i/E_q \tag{10}$$

Then one can show that all Lorentz scalars involving final states only can be represented in terms of x_i and particle masses. In-fact, we only need to check $(k_1 + k_2)^2$, $(k_2 + k_3)^2$ and $(k_3 + k_1)^2$. For instance, we can have

$$(k_1 + k_2)^2 = (q + k_3)^2 = q^2 + m_3^2 - 2q \cdot k_3 = s(1 - x_3) + m_3^2 \tag{11}$$

To simplify the integral, we first integrate out K3 with spatial delta function that restricts $K_3 = k_1 + k_2$ such that

$$\int d\Pi_3 = \int \frac{d^3k_1, d^3k_2}{(2\pi)^6 2E_1, 2E_2, 2E_3} 2\pi\delta(E_q - E_1 - E_2 - E_3) \tag{12}$$

The integral measure can be rewritten as

$$d^3k_1 d^3k_2 = k_1^2 k_2^2 dk_1 dk_2 d\Omega_1 d\Omega_{12} \tag{13}$$

where $d\Omega_1$ is the spherical integral measure associated with d_3k_1, and $d\Omega_{12}$ is the spherical integral of relative angles between k_1 and k_2. The former spherical integral can be directly carried out and result in a factor 4π. To find the integral with

$$d\Omega_{12} = d\cos\theta_{12} d\pi_{12} \tag{14}$$

we make use of the remaining delta function which can be rewritten as,

$$\delta(E_q, -E_1 - E_2 - E_3) = \frac{e_3}{k_1 k_2}\delta\left(\cos\theta_{12} - \frac{E_3^2 - k_1^2 - k_2^2 - \mu^2}{2k_1k_2}\right) \tag{15}$$

by means of

$$E_3 = \sqrt{k_1^2 + k_2^2 + 2k_1^2 k_2^2 \cos\theta + \mu^2} \tag{16}$$

Thus we have

$$\int d\Omega_1 d\Omega_{12} = \delta(E_q, -E_1 - E_2 - E_3) = \frac{8\pi^2 E_3}{k_1 k_2} \tag{17}$$

Now, using

$$k_1 dk_1 = E_1 dE_1 \quad and \quad k_2 dk_2 = E_2 dE_2 \tag{18}$$

we have

$$\int d\Pi_3 = \int \frac{d^3k_1 d^3k_2 k_1^2 k_2^2}{8(2\pi)^5 E_1 E_2 3} \cdot \frac{8\pi^2 E_3}{k_1 k_2} = \frac{1}{32\pi^3}\int dE_1 dE_2 = \frac{1}{128\pi^3}\int dx_1 dx_2 \tag{19}$$

To determine the integral region for $m_1 = m_2 = 0$ and $\mu_1 = \mu_2$, we note that there are two external cases: $\mathbf{k}_1$ and $\mathbf{k}_2$ are parallel and anti-parallel. In former case, we have

$$E_q = E_1 + E_2 + E_3 = E_1 + E_2 + \sqrt{(E_1 + E_2)^2 + \mu^2} \tag{20}$$

which yields

$$2E_q(E_1+E_2)=E_q^2-\mu^2 \tag{21}$$

which gives

$$(E_q-2E_1)(E_q-2E_2)=\mu^2 \tag{22}$$

These two boundary cases can be represented by x_i variables as

$$x_1+x_2=1-\frac{\mu^2}{s}, \qquad (1-x_1)(1-x_2)=\frac{\mu^2}{s} \tag{23}$$

The integral thus goes over the region bounded by these two curves.

III. Difference Cross Section

Now we calculate the differential cross section for the process electron plus positron into quark-qurk-antigluon to lowest order in α and αg. First, the amplitude is

$$iM=Q_f(-ie)^2(-ig)\epsilon_v^*(k_3)\bar{v}(k_1)\left[\gamma^v\frac{i}{k_1+k_2}\gamma^\mu-\gamma^\mu\frac{i}{k_1+k_2}\gamma^v\right]v(k_2)\frac{i}{q^2}\bar{v}(p_2)\gamma_\mu u(p_1) \tag{24}$$

Then, the squired amplitude takes the form

$$\begin{aligned}\frac{1}{4}\sum|\,iM\,|^2&=\frac{Q_f^2g^2e^4}{4_s^2}(-g_{v\sigma})T_r(\gamma_\mu p_1)\gamma_p P_2)\\&\times T_r\left[\left(\gamma^v\frac{i}{k_1+k_3}\gamma^\mu-\gamma^\mu\frac{i}{k_2+k_3}\gamma^v\right)k_2\left(\frac{i}{k_1+k_3}\gamma^\sigma-\gamma^\sigma\frac{i}{k_2+k_3}\gamma^p\right)k_1\right]\\&=\frac{4Q_f^2g^2e^4}{3_{s^2}}(8p_1\cdot p_2)\times\left[\frac{4(k_1\cdot k_2)(k_1\cdot k_2+q\cdot k_3)}{(k_1+k3)^2(k_2+k_3)^2}+\left(\frac{1}{(k_2+k_3)^4}+\frac{1}{(k_2+k_3)^4}\right)\right]\\&\times[2(k_1\cdot k_3)(k_2\cdot k_3)-\mu^2(k_1\cdot k_2)]\end{aligned} \tag{25}$$

We have used the trick described well in [1] when getting the last equal sign. Now we rewrite the quantities of final-state kinematics in terms of x_i and set $\mu \to 0$ we obtain

$$\begin{aligned}\frac{1}{4}\sum|\,iM\,|^2&=\frac{2Q_f^2g^2e^4}{3_{s^2}}(8p_1\cdot p_2)\left[\frac{2(1-x_3)}{(1-x_1)(1-x_2)}+\frac{1-x_1}{1-x_2}+\frac{1-x_2}{1-x_1}\right]\\&=\frac{8Q_f^2g^2e^4}{3_{s^2}}\frac{x_1^2-x_2^2}{(1-x_1)(1-x_2)}\end{aligned} \tag{26}$$

Thus, the differential cross section, with 3 colors counted, reads

$$\begin{aligned}\frac{d\sigma}{dx_1dx_2}|_{com}&=\frac{1}{2E_{p_1}2E_{p_2}\,|\,v_{\vec{P_1}}-v_{\vec{P_2}}\,|}\frac{s}{128\pi^3}\left(\frac{1}{4}\sum|\,iM\,|^2\right)\\&=\frac{4\pi\alpha^2}{3_s}\cdot 2Q_f^2\cdot\frac{\alpha g}{2\pi}\frac{x_1^2-x_2^2}{(1-x_1)(1-x_2)}\end{aligned} \tag{27}$$

where we have used the fact that the initial electron and positron are massless, which implies that

$$2E_{\vec{p_1}}=2E_{\vec{p_2}}=\sqrt{s} \quad \text{and} \quad |\,v_{\vec{P_1}}-v_{\vec{P_2}}\,|=2 \tag{28}$$

in the com frame.

IV. The Average Square Amplitude

Now we re-evaluate the average square amplitude, with μ kept non-zero in Equ. (25). The result reads

$$\frac{1}{4}\sum | iM |^2 = \frac{8Q_f^2 g^2 e^4}{3_s} F(x_1, x_2, \mu^2/s) \tag{29}$$

where

$$F\left(x_1, x_2, \frac{\mu^2}{s}\right) = \frac{2\left(x_1 + x_2 - 1 + \frac{\mu^2}{s}\right)\left(1 + \frac{\mu^2}{s}\right)}{(1-x_1)(1-x_2)} + \left[\frac{1}{(1-x_1)^2} + \frac{1}{(1-x_2)^2}\right]\left((1-x_1)(1-x_2) - \frac{\mu^2}{s}\right) \tag{30}$$

The cross section, then, can be gotten by integrating over dx_1, dx_2 such that

$$\begin{aligned}\sigma\left(e^+e^- \longrightarrow q\bar{q}g\right) &= \frac{1}{2E_{\vec{p_1}} 2E_{\vec{p_2}} \mid v_{\vec{p_1}} - v_{\vec{p_2}} \mid} \frac{s}{128\pi^3} \int dxdy \left(\frac{1}{4}\sum | M |^2\right) \\ &= \frac{4\pi\alpha^2}{3_s} \cdot 3Q_f^2 \cdot \frac{\alpha g}{2\pi} \int_0^{1-\frac{\mu^2}{s}} dx_1 \int_{1-x_1-\frac{\mu^2}{s}}^{1-\frac{t}{s(1-x_1)}} dx_2 F\left(x_1, x_2, \frac{\mu^2}{s}\right) \\ &= \frac{4\pi\alpha^2}{3_s} \cdot 3Q_f^2 \cdot \frac{\alpha g}{2\pi} \left[\log^2 \frac{\mu^2}{5} + 3\log\frac{\mu^2}{5} + 5 - \frac{1}{3}\pi^2 + O(\mu^2)\right]\end{aligned} \tag{31}$$

V. Feynman Parameters

It is straightforward to finish the integration over Feynman parameters in our preliminary computation, yielding

$$F_1(q^2 = 5) = Q_f^2 - \frac{Q_f^2 \alpha g}{4\pi}\left[\log^2\frac{\mu^2}{5} + 3\log\frac{\mu^2}{5} + \frac{7}{2} - \frac{1}{3}\pi^2 - i\pi\left(2\log\frac{\mu^2}{5} + 7\right) + O(\mu^2)\right] \tag{32}$$

Then the cross section, to the order of αg, is given by

$$\sigma\left(e^+e^- \longrightarrow q\bar{q}\right) = \frac{4\pi\alpha^2}{3_s} \cdot 3Q_f^2 \left\{1 - \frac{\alpha g}{2\pi}\left[\log^2\frac{\mu^2}{5} + 3\log\frac{\mu^2}{5} + \frac{7}{2} - \frac{1}{3}\pi^2 - O(\mu^2)\right]\right\} \tag{33}$$

Results

Finally combining Equ. (31) and Equ. (33) we arrive at our final result as:

$$\sigma\left(e^+e^- \longrightarrow q\bar{q} + q\bar{q}g\right) = \frac{4\pi\alpha^2}{3_s} \cdot 3Q_f^2 \left[1 + \frac{3\alpha g}{4\pi}\right] \tag{34}$$

It is worth noting that all divergent terms as $\mu \to 0$ cancel in this expression.

Conclusion

In conclusion, we wish to stress that in order for one to confront the infrared divergence in this computation and better understand such an expression, one must do some work in order to simplify it. This, as seen from above, is achieved by extracting such an expression and evaluating the divergent part of the integral. Thereafter a compulsory result can be arrived at.

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