Original Article



The Universe in Resonance: Mathematics, Music and Health in Balance

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Abstract

This paper presents the succession of the digital roots of the numbers 1 to 144^[1] and reveals numerical patterns that match the Hertz frequencies of musical notes in Solfeggio scales. This result, which connects apparently different subjects such as mathematics and music, has potential implications for science and technology.

In this work the digital roots of the numbers from 1 to 144 are calculated. The sequences of the digital roots are analysed to identify repeating patterns. After that, the Hertzian frequencies of the musical notes in the Solfeggio scales are related. Finally, repetitive numerical patterns were found in the succession of the digital roots that matched the Hertzian frequencies of the musical notes in the Solfeggio scales.

This result suggests a deep connection between mathematics and music. Mathematics, in its apparent abstraction, manifests itself in the harmony of sound. This connection could have implications for the understanding of the universe and human perception.

In addition, it may have applications in technology: The numerical patterns of the digital roots could be used to develop new, more efficient and secure communication technologies; as well as to generate energy, so that the Solfeggio frequencies could be used to resonate with the natural frequencies of the Earth's ionospheric layer, amplifying the energy available for electricity production.

This line of study opens a new chapter in the history of science and technology. Human inventiveness and creativity, inspired by the beauty of mathematics and music, can lead to unimaginable breakthroughs.

Keywords: Digital roots, Nikola Tesla, frequencies, Solfeggio, mathematics, music, science, technology, universe in resonance, energy.

1. Theoretical Foundations

In the search for connections between different areas of knowledge, mathematics and music have been linked throughout history. This paper presents an interesting analysis of the succession of the digital roots of the numbers from 1 to 144, which reveals repetitive numerical patterns that coincide with the Hertzian frequencies of the musical notes in the traditional and extended Solfeggio scales.

1.1. Mathematical pattern

In Mathematics, a pattern is a repeated arrangement of numbers, shapes, colours and so on. The Pattern can be related to any type of event or object. If the set of numbers are related to each other in a specific rule, then the rule or manner is called a pattern. Sometimes, patterns are also known as a sequence, and therefore, patterns are finite or infinite in numbers.

That is, a pattern is a series of constant and identifiable variables within a larger data set $^{[2]}$.

Science is based on observation and data analysis to expand knowledge. In this process, the identification of patterns is fundamental to understanding the structure of the universe and formulating laws that explain its behaviour.

A mathematical pattern is defined as an identifiable regularity in a sequence of elements, whether they are numbers,

geometric figures or any other type of data. Its study allows the discovery of relationships and the formulation of predictions, laying the foundations for the development of science and technology.

The study of mathematical patterns is a tool for discovering the underlying order in nature, formulating laws that explain its behaviour and developing new technologies that improve human life.

1.2. Digital root of a number

In mathematics, the decimal system is based on the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, giving rise to the quantitative principle on which education and science, in general, are based. This has allowed science to advance towards the concept of infinity with multiple theories and laws, leaving aside the qualitative aspects of numbers, i.e. their "essence".

In the search for mathematical patterns, the concept of the digital root of numbers is very important, which, in a simple but very powerful way, allows us to obtain the essence of their nature.

One of these important qualitative aspects corresponds to the digital root of a positive integer number n which is defined as the single digit obtained by repeatedly adding the digits of n until a single digit result is obtained. It is denoted by RD(n).

Example: RD (14) = $1 + 4 \rightarrow 5$.

That is, the digital root or repeated digital sum of a positive integer in any numbering base is the single-digit value obtained by an iterative process of digit addition, where in each iteration the result of the previous iteration is used to calculate the digit sum. The process continues until a single digit number is obtained. See ^[1].

For example, in base 10, the digital root of the number 16 is $1+6 \rightarrow 7$ and that of 32 is $3+2 \rightarrow 5$.

For a number like 12345 its digital root RD (12345) is $1+2+3+4+5 \rightarrow 15$ and if we iterate it again $1+5 \rightarrow 6$.

In base 10, this is equivalent to taking the remainder of the division by 9 (except when the digital root is 9, where the remainder of the division by 9 will be 0).

This would formally be the digital root RD for a positive integer n with base b > 1, defining the sum of its digits as:

$$N_b(n) = \sum_{i=0}^{k-1} RD_i$$

Where $K = [log_b(n)]+1$ which corresponds to the number of digits of the base b number, remaining:

$$RD_i = \frac{n \cdot mod \ b^{i+1} - n \cdot mod \ b^i}{b^i}$$

valid for $0 \le n < b$. As an example, the digital root of the number 2024 in base 9 is calculated according to the above definitions: Example: For n=2024, N₉(2024) \rightarrow 2+0+2+4 \rightarrow 8.

The digital root allows us to visualize the position of the natural numbers with respect to a multiple of 9, so that, if we have a natural number and we subtract its digital root, we will obtain a multiple of 9. Example: So $2024-8=2016 \rightarrow$ is a multiple of 9, since 224*9=2016.

As we will see below, all numbers indicate with the digital root how far they are from the equilibrium of 9.

It is known that the only possible values of the digital root of the prime numbers other than 3 are: 1, 2, 4, 5, 7, 8. It is necessary to observe that 3, 6 and 9 do not appear.

1.3. Ulam's spiral

The mathematician Stanislaw Ulam established in 1963 a simple method of representing prime numbers by a graph, which shows a pattern. Ulam' s spiral consists of arranging a grid of numbers, starting with 1 in the centre and arranging them in a spiral from there. He then circled the prime numbers and obtained a pattern in which the prime numbers were aligned in diagonal lines.



1.4. Map of multiplication

Nikola Tesla, a visionary inventor and engineer, not only made his imprint in the field of electricity and magnetism, but also delved into the field of mathematics. Among his most intriguing ideas is the spiral map of multiplication of 12/12/1912. It is a spiral diagram with 12 positions in which the natural numbers from 1 to 144 are placed and in which their multiplication relationships and prime numbers are explained. Currently this Tesla map is available on the Internet, and it would be interesting to review the image of it (see ^[4]).



Fig. 2: Tesla's multiplication map: Prime and composite numbers as a self-organizing system", Nikola Tesla ^[4].

In this map, Nikola Tesla found a geometric pattern for each repeating number:

- 1. The number 2 and 10 act as "doublers" that alternate between doubling prime positions next to and between them. Multiples of 2 on the map result in a regular hexagon.
- 2. The number 3 multiplies itself through the system as a perfect square that bounces between 3, 6 and 9 to 12. All multiples of 3 are in these positions. Multiples of 3 give rise to a square.
- 3. The number 4 multiplies through the system as an equilateral triangle, jumping between positions 4, 8 and 12. All multiples of 4 jump in these positions and give rise to a triangle.
- 4. The number 6 multiplies itself in the system as a straight line. It jumps back and forth between positions 6 and 12.
- 5. The number 5 is the first main position. It works like a star in some respects by jumping back and forth through the system in a counterclockwise direction.
- 6. The number 7 is the second principal position. It mirrors the trajectory of 5 touching each position exactly opposite by crossing the trajectory of 5 clockwise.
- 7. The number 11 is the top principal position. It cascades to the left and turns the system upside down. An inverse spiral is generated.
- 8. The 1 or 13 is the upper right position. It mirrors the 11.

1.5. Musical scale of Solfeggio

Solfège comes from the French "solfège" and the Italian "solfeggieto". Both derive from the names of two of the notes used: sol and fa. Solfège is a musical training method used to teach intonation with the voice during the reading of a score. It consists of intoning while reciting the names of the notes of the melody, respecting the durations of the notes, to keep the rhythm. Solfège is the musician's fundamental tool for exploring sounds through musical scales. These ordered structures of notes form the basis of melody and harmony for the expression of emotions and connection with the audience ^[5].

A musical scale is a succession of sounds or notes arranged according to a particular system or mode. These notes follow each other regularly by joint degrees, i.e. by tones and semitones, in an ascending or descending direction within the extension of an octave (interval separating two sounds whose frequency is in a ratio of 2 to 1). Musical scales are the basis of melody and harmony, by means of sets of notes arranged in ascending or descending order that determine the tonality and character of a piece of music ^[6].

Solfeggio musical scale is a system of intonation using six notes: do, re, mi, fa, sol, la, as it is well-known. These notes are associated with syllables that facilitate the reading and learning of music.

The origin of the Solfeggio frequencies can be found in the 11th century, with the music theorist Guido d'Arezzo who developed an ascending scale of six notes as follows: Ut, Re, Mi, Fa, Sol, and La. The names were taken from the first verse of the Latin hymn "*Ut quéant laxis*", where the syllables fall on their corresponding scale degree (except "Si"), see ^[7].



Fig. 3: Figure from the Latin hymn of St John ^[7].

"Ut" was changed in 1600 in Italy to the open syllable Do, at the suggestion of the musicologist Giovanni Battista Doni, and Si (from the initials of "Sancte Iohannes") was added to complete the diatonic scale.

The Solfeggio frequencies are as follows and have been related to the following improvements for people in different schools of sound therapy throughout history, as well as to the chakra system of oriental culture as well:

- UT 396 Hz Releasing guilt and fear The root chakra.
- RE 417 Hz Undoing situations and facilitating change
- The sacral chakra

- MI 528 Hz Transformation and miracles (DNA repair)
 The Solar Plexus Chakra
- FA 639 Hz Connection / Relationships The heart chakra
- SOL 741 Hz Awakening intuition The throat chakra
- LA 852 Hz Returning to the spiritual order The third eye chakra
- 963 Hz The crown chakra.

But in Guido d'Arezzo's time, there were no standard tuning tools, nor precise clock measurements to know the Solfeggio frequencies. The Hertz unit was established in 1960 by the International Electrotechnical Commission.

So how could these notes be measured in hertz (or Hertz) frequencies when there was no hertz measurement, no use of the second, no agreement on tuning and no method of measurement?

This work relates the sequence of digital roots of numbers represented in a simplified form according to Nikola Tesla's multiplication spiral to the frequencies of the notes of the traditional and extended Solfeggio musical scale.

2. Methodology

Based on the above theoretical concepts, this paper applies the digital roots of numbers to the Tesla's multiplication spiral to obtain numerical patterns and better understand its design. The digital root of each number from 1 to 144 were computed. Subsequently, the sequences of digital roots were analysed to identify repeating patterns. Finally, the sequences were compared with the hertz frequencies of the musical notes in the Solfeggio scales.

In this way, from a simplification of the spiral by means of a concentric ring diagram, was obtained the graphical feeling generated by a spreadsheet:



Fig. 4: Eigen-diagram of the sequence of natural numbers in ring graph.

In this diagram it can be seen that the prime numbers are found in the rings, painted in blue, starting from 1, 5, 7 and 11. These root prime numbers generate the rest of the derived prime numbers by adding 12 to them, with the exceptions already indicated by Nikola Tesla, so that neither their squares nor the multiples of their interactions are included when they are multiplied together. For example, 5x5=25 is not prime, nor is 5x7=35 prime.

In fact, being a logarithmic spiral, its distribution is threedimensional. Continuing with this research, numerical patterns were sought by applying the digital roots to the previous simplified diagram, obtaining as a result:



Fig. 5: Eigen-diagram of the sequence of digital roots of the natural numbers in ring graph.

In this way, the 12 patterns for each sector of the diagram were obtained. As this is an organised system of numbers with logical organisation, it can be linked to music and its scales.

In the study of the numbers in the figure above, Nikola Tesla in his spiral of multiplication did not consider the importance of their digital roots, as well as the numerical patterns they generate and their relationships with other subjects such as music.



Fig. 6: A figure explaining the digital root sequences of the numbers of Nikola Tesla's multiplication map.

The patterns obtained and their relationship with frequencies are:

Table I: Information about the patterns and frequencies.

Numbers	EXPANSIÓN (Outward)	IMPLOSION (Inward)	FRECUENCIES ' (Hz)	MUSICAL NOTES (Traditional Solfeggio Scale)	FREQUENCIES AND BENEFITS FOR THE HUMAN BODY (Chakra)
1	1-4-7	7-4-1	741		Throat
2	2-5-8	8-5-2	852		Third Eye
3	3-6-9	9-6-3	963		Crown
4	4-7-1	1-7-4	174		Earth
5	5-8-2	2-8-5	285		Healing
6	6-9-3	3-9-6	396	Do	Root
7	7-1-4	4-7-1	417	Re	Sacrum
8	8-2-5	5-2-8	528	Mi	Solar Plexus
9	9-3-6	6-3-9	639	Fa	Heart
10	1-4-7	7-4-1	741	Sol	Throat
11	2-5-8	8-5-2	852	La	Third Eye
12	3-6-9	9-6-3	963	Si	Crown

(*) Remark: The current musical notes do not correspond to the old traditional Solfeggio scale of sacred music.

3. Results

As a result of this work, repetitive numerical patterns were found in the succession of digital roots. These patterns coincided with the Hertzian frequencies of the musical notes of the Solfeggio scale and with the natural frequencies of the human being in his different chakras. From the numerical patterns obtained by applying the digital root of the sequence of numbers, these can be related to the frequencies in cycles per second of the notes of the Solfeggio scales resulting in the following, once ordered correctly:



Fig. 7: A figure explaining the sequences of digital roots of the numbers of Nikola Tesla's multiplication map and relates them to the frequencies of the traditional Solfeggio musical scale.

Table II: Sum of frequencies to obtain their horizontal and vertical digital roots.

Horizontal sums	Result		Digital Root
963=	963	9+6+3>15+3=18>9	9
852+174=	1026	1+0+2+6>9	9
741+285=	1026	1+0+2+6>9	9
963+396=	1359	1+3+5+9>9+9>18>9	9
852+417=	1269	1+2+6+9>3+6+9>9+9=18>9	9
741+528=	1269	1+2+6+9>3+6+9>9+9=18>9	9
639=	639	6+3+9>9+9=18>9	9
Vertical sums	Result		Digital Root
396=	396=	9+6+3>15+3=18>9	9
285+417=	702=	1+0+2+6>9	9
174+528=	702=	1+0+2+6>9	9
963+639=	1602=	1+0+3+5>9	9
852+741=	1593=	1+2+6+9>3+6+9>9+9=18>9	9
741+852=	1593=	1+2+6+9>3+6+9>9+9=18>9	9
963=	963=	9+6+3>9+9=18>9	9

These results suggest a fundamental connection between mathematics and music, as it is well-known. The seemingly random succession of digital roots of numbers hides a mathematical organization that is reflected in musical harmonies. This connection could have implications for the understanding of the universe and human perception.

Moreover, this 12-note musical scale can be arranged with semitones between each musical note by means of a 24-sector diagram, so that by calculating the digital roots of the frequencies, they correspond to 3, 9, 6 in a repetitive manner as a new pattern.





In the following figure you can see the pattern of numbers 3, 9, 6 that is repeated when calculating the digital roots of the frequencies of the notes and the semitones between them.



Fig. 8: A scheme explaining the digital root sequences of the numbers of the Nikola Tesla's multiplication map and relates them to the frequencies of the musical notes and their semitones of the musical scale.

As previously pointed out, the Solfeggio frequencies are an ancient musical scale that have been used in sacred music and do not correspond to the current musical frequencies. But this new musical scale, that I would like to call as "Carrasco's mathematical-musical scale", is based on the traditional solfeggio scale of 7 musical notes, but extended to 12 notes with 12 semitones between them (obtained as the average between two notes) has some unique particularities:

- 1. **Constant sum of digital roots**: the digital roots of the frequencies of the 12 notes, arranged in a circle, always add up to 9.
- 2. **Repetitive sequence of notes and semitones**: The digital roots of the semitone intervals between the notes and the musical notes form a repetitive sequence of 3, 9, 6.

These mathematical properties suggest an underlying harmony in this "perfect" scale that could have significant implications in various scientific fields and opens new possibilities in the understanding of music, sound and the benefits for the human body.

4. Possible Scientific Implications and Technological Applications

The digital roots of numbers could be a mathematical code that hides information about nature and music; so, the musical notes could be a representation of the natural harmonies of the Earth. In that case, the Solfeggio frequencies, resonating with the Earth, could have effects on human consciousness and on people's well-being and health.

This connection could have profound scientific implications for the understanding of the nature and perception of sound; with possible applications in areas such as music composition, sound therapy and music education; highlighting the following:

- Mathematics: This new scale could be a mathematical model for understanding harmony and the structure of sound.
- Physics: The connection between the digital roots of Solfeggio numbers and frequencies could provide new insights into the wave nature of matter and energy, and the search for a unified theory of physics. The relationship between the frequencies of the perfect scale and the constant sum of their digital roots could reveal new physical properties of sound and vibration.

- **Biology**: Solfeggio frequencies, which match the natural resonances of the human body, could have applications in medicine and sound frequency therapy.
- Neuroscience: The repetitive sequence of 3, 9, 6 in semitones could have an impact on musical perception and brain response to music. This scale could have applications in bioacoustics, studying communication and sound perception in animals.
- Technology: New communication, information storage and data processing technologies could be developed based on the mathematical and musical properties of the digital roots of numbers.
- Art: Musical composition could benefit from the application of these numerical patterns, creating new harmonies and melodies.
- Education: The teaching of mathematics and music could be creatively integrated, using this finding as a starting point. This new scale could be a pedagogical tool to teach music in a more intuitive and understandable way.
- **Music**: This new musical scale could be used to compose music with new sonorities and harmonies, expanding the creative possibilities of musicians.
- **Therapy**: This finding could have applications in music therapy, using its properties to improve mental and physical health.
- **Technology**: The perfect scale could be used in the development of new audio and sound processing technologies.

Regarding possible industrial applications, the following could be studied:

- Wireless communications: The numerical patterns of the digital roots could be used to develop new, more efficient and secure communication technologies.
- **Electric power generation**: The frequencies of this new scale could be used to resonate with the natural frequencies of the Earth's ionosphere layer, amplifying the energy available for power generation.

5. Interpretations of the results

Solfeggio frequencies produce more positive effects on the body than any other sound or tone. And it is due to the Schumann resonance. In 1952, the German physicist Winfried Otto Schumann mathematically documented the electromagnetic resonances that exist between the Earth's surface and the ionosphere, the electrically charged part of the atmosphere. He found that these electromagnetic waves, which originate from lightning discharges, resonated at a low frequency between 7.86 and 8 Hz. He determined that this frequency was in essence the heartbeat of the Earth. It has since become known as the Schumann resonance in honour of its founder.

Schumann's successor, Herbert Konig, took the research a step further. He studied the connection between Schumann resonances by comparing EEG recordings with terrestrial electromagnetic fields and found that they coincide with different levels of human brain activity. Konig found that the resonances matched five distinct brain wave states: delta, theta, alpha, beta, and gamma. These states are those that occur naturally during everyday activities, from sleeping to creating to learning. Subsequent research supports Konig's findings confirming the incredible similarities between Schumann resonance and brain activity. In addition, other research indicates that the low frequency of the Schumann resonance provides synchronization for higher brain function. The latter have such positive effects because they resonate in harmony with the 8 Hz Schumann resonance. In musical terms, the frequencies are derived from 8 Hz and ascend octave by octave in the musical scale until the note C vibrates at the frequency of 256 Hz and the note A vibrates at 432 Hz. When music is tuned to harmonize with this frequency, it is called scientific tuning ^[8].

The Solfeggio frequencies include six different tones among others. Let's take a closer look at each frequency and the unique healing effects on the body and mind.

The connection between digital roots, Solfeggio frequencies and the natural resonances of the human body opens a new horizon of possibilities in science and technology. This connection, which brings together mathematics, music, and biology, could have a significant impact on the well-being and health of people and their natural environment. However, further research is needed to understand and take full advantage of these connections. In the Solfeggio Scale there are six main frequencies, each with its own energy and benefits to the human body. The main frequencies and their benefits with the system of chakra are detailed below (See ^[9]):

174 Hz: the frequency of the earth

The frequency of 174 Hz is said to be in resonance with the energy of the earth. Listening to this frequency can help reduce stress and anxiety, improve sleep quality, and help balance the root chakra. The root chakra is located at the base of the spine and is associated with security and stability. Listening to the 174 Hz frequency can help improve grounding and a sense of security.

285 Hz: the healing frequency

The 285 Hz frequency has healing properties and helps relieve pain and inflammation. It is also used to balance the heart chakra and improve energy circulation. It is said that this frequency can help improve self-esteem and promote self-love.

396 Hz: the frequency of release

The 396 Hz frequency is used to release emotional and psychological blockages and encourage the release of negative patterns. This frequency is also known as the "unblocking frequency" and is said to help release guilt, fear, and anxiety. It is believed that listening to the 396 Hz frequency can help reduce stress levels and promote relaxation.

417 Hz: the frequency of transformation

The 417 Hz frequency is used to promote transformation and change. This frequency helps to release negative patterns and bring positive changes in life. It is also used to balance the sacral chakra, which is in the lower abdomen and is associated with creativity and sexuality.

528 Hz: the DNA repair frequency

The frequency of 528 Hz is the DNA repair frequency and is believed to have a positive effect on the cells of the human body. This frequency is said to help repair damaged DNA, improve cell function, and stimulate tissue regeneration. It is also used to balance the third eye chakra, which is in the centre of the forehead and is associated with intuition and perception.

639 Hz: the frequency of connection

The frequency of 639 Hz is used to foster connection with others and with oneself. This healing frequency helps to improve interpersonal relationships and promote harmony and balance in relationships. It is also used to balance the throat chakra, which is associated with communication and self-expression.

741 Hz: the frequency of expression and resolution.

The frequency of 741 Hz is used to promote expression and conflict resolution. This healing frequency helps to open communication and facilitate the expression of repressed emotions. It is also used to balance the third eye chakra and throat chakra, promoting mental clarity and effective communication.

852 Hz: the frequency of intuition and spiritual awareness.

The frequency of 852 Hz is considered a healing frequency of spiritual activation. This frequency is believed to help awaken intuition and raise spiritual awareness. It is also used to balance the crown chakra, which is located at the top of the head and is associated with connection to the divine and expansion of consciousness.

963 Hz: the frequency of pineal gland activation.

The frequency of 963 Hz is associated with the activation and stimulation of the pineal gland, also known as the "third eye". This healing frequency helps to open perception and intuition, facilitating access to higher dimensions of consciousness. It is also used to balance the crown chakra and to promote unity and connection with the universe.

The convergence of mathematics, music and biology in this new field opens a path to a more harmonious and sustainable future. Research in this field can generate unimaginable advances in the well-being and health of people and their natural environment.



Fig. 9: Figure indicating the chakra system with the frequencies of the traditional musical scale of Solfeggio ^[7].

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