

Deterministic Finite State Model of Heart Cycle: Modeling and Simulation

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Abstract

Heart cycle model explores the deterministic finite state of heart cycle whereby the transition of blood is dependent of the state of the source and the input. Blood flowing in human body is a series of complex networks working together in a logic circuit. This research describes the complex mechanisms of heart cycle in relation to the inter-relationship between the chambers and valves, in a logical and one dimensional structure. Logical operators and tristate buffer are used to map the states of the chamber and valves as it relaxes/contracts and open/close respectively, in truth tables. Mathematical analysis is used to model the state and outflow behavior of heart cycle in discrete time.

Keywords: heart cycle, blood flow, valve disease, finite state, discrete time.

1. Introduction

Modeling and simulation is diverse as there exists numerous fields of its application, its concepts vary from model description, simplification, validation, simulation and exploration and is not specific to any particular discipline (Zeigler, Praehofer, & Kim, 2000, 2nd Edition). In this study, we look at the underlying and exploitable unity in the functions of the heart chambers and valves; adopt the use of logical and algebraic analysis in representation of the observed inter-relations of the heart chambers and valves; and formalize the concepts involved in relation to the state and output behavior in discrete time.

According to (National Institute of Health, Page Updated: 2015) valves defects can be acquired later in life or may develop before birth (congenital heart valve disease). The valve problems can occur either individually or with other congenital heart defects. Mathematical models of biological systems range from continuous to discrete based on the representation of the status of the system's components and from deterministic to stochastic based on their incorporation of randomness and noise (Istvan, Juilee, Song, Ranran, & Reka, 2008). Combination of improved numerical algorithms and increased processing power has enabled the use of computer models to understand vascular blood flow (Taylor, Hughes, & Zarins, 1998).

Here, mathematical analysis was used to model the infinite series of the heart cycle by describing the states of the valve and chamber using discrete time models. We first described stepwise execution for a global discrete time analysis of the heart cycle in its ideal state due to the fact that it beats at 72seconds in a minute hence for every 0.83 seconds the heart is assumed to beat once. This one beat is both diastole and systole, of which the heart valves inherit the same properties. Binary approach is also used to simulate the performance and inter-relationship of the heart valves and chambers (atriums and ventricles) in flow of blood. Blood flowing from one chamber to the other is deterministic for an ideal functioning heart although it could change to stochastic due to generic or acquired disease in the valves, chambers or blood. The binary representations and truth tables represent the state of the heart both in its optimal performance and failure.

The relationship between the model variables and disease is illustrated, where the type of disease is directly dependent on the various states of the valves and chambers.

2. Related Works

Our study on previous related works was on both visualizations and numerical analysis of the heart chambers and valves and blood flow. Boolean networks are eminently suited at mapping biological observations and hypotheses into a mathematical formalism (Istvan, Juilee, Song, Ranran, & Reka, 2008). (Charles, 2004) simulated blood flow and carried out a numerical analysis of blood flow in the heart. Study of numerical simulation and experimental validation of blood flow in arteries with structured-tree outflow conditions (Mette, Charles, Won, Erik, Ali, & Jesper, 2007). Study on mesoscopic simulations of systolic flow in the human abdominal aorta (Artoli, Hoekstra, & Slood, 2005) who observed that lattice Boltzmann method is a successful mesoscopic solver to time dependent blood flow in an arterial system and a good alternative for computational hemodynamics (Lilit, Hoekstra, Jeays, Lawford, Hose, & Slood, 2009). Numerical simulation of blood flow in the left ventricle and aortic sinus using magnetic resonance imaging and computational fluid dynamics (Mir-Hosseini, Nasser, Hamid, Ali, Oscar, & Frangie, 2014).

Intelligent system for diagnosis of the heart valve disease using wave packet neural networks by (Ibrahim, Ahmet, & Erdogan, 2003), similarly, (Comak, Arslan, & Turkoglu, 2007) developed a decision support system based on support vector machines for diagnosis of the heart valve diseases. In the study by (Uguz, A Biomedical System on Artificial Neural Network and Principal Component Analysis for Diagnosis of the Heart Valve Diseases, 2010) biomedical-based decision support system was developed for the classification of heart sound signals. An extension of his work (Uguz, Adaptive Neuro-fuzzy Inference System for Diagnosis of the Heart Valve Diseases using Wavelet transform with Entropy, 2011). Biomedical system based on hidden Markov model for diagnosis of the heart valve disease classified valve states into normal and abnormal valve (Uğuz, Arslan, & Türkoğlu, 2007) and was an extension from (Turkoglu, Arslan, & Ilkay, 2002) whose classification of abnormal data included all diseases related to aortic and mitral valves such as aortic insufficiency and stenosis, and mitral insufficiency and stenosis. Heart Valve Disease Classification using Neural Network by (Shelke & Baru, 2013) was on aortic stenosis, mitral regurgitation, mitral stenosis and aortic regurgitation where heart sound wave was segmented into S1, S2 systolic murmur and diastolic murmur then the segments features was input to artificial neural network to classify disease.

This paper presents classification of the heart valve disease in all the four chambers of the heart using binary representation where 0 depicts a disease and 1 depicts the true state of the chamber. The state of the heart is further represented as full (0) and not full(1) in representation of diastolic and systolic phase, similarly, the heart valves was represented as open (1) and closed (0). The combination of representative variables of the chamber (atria and ventricles) states and valves states demonstrated inflow and outflow of deoxygenated and oxygenated blood. Further, a mathematical analysis is done to understand the behavior of the states as it relates to the chambers input and output.

3 Methodology and Analysis

This paper uses one dimensional representation to develop a logical model which describes the flow of blood in the human body, logical function (AND, &) is used to model the inputs using binary variables to truth tables. A tristate buffer is used as a switch to model the states of the valves which is either closed or open.

3.1 Logic Circuit Model of Heart Cycle

The heart has mainly two (2) states:

- a) The diastole - when the heart relaxes as blood flows inside the heart
- b) Systole – when the heart contracts to pump blood out

We therefore generalize the whole heart blood flow in two binary variables: Chambers (Atriums and Ventricles); Not full = 0 and Full =1

Valves; Open = 1 and Closed = 0

It is important to note that some states cannot function without the other, for example, we cannot model contract state without relax state, the vice versa is true though. An example in the real world by using rain as the sample: - *If it rains you get wet, and to avoid being wet you use an umbrella. It can rain and you do not get wet, but you cannot get wet without rain.*

This paper further focus on the four chambers of the heart, that is, the atriums and their adjoining valves likewise the ventricles with their adjoining valves, to describe the process of the deoxygenated and oxygenated blood flow. We will realize as we proceed further in our analysis that to maintain a forward motion of blood flow through the ventricles there has to be co-ordination among the two valves that intersect with the right and the left ventricles.

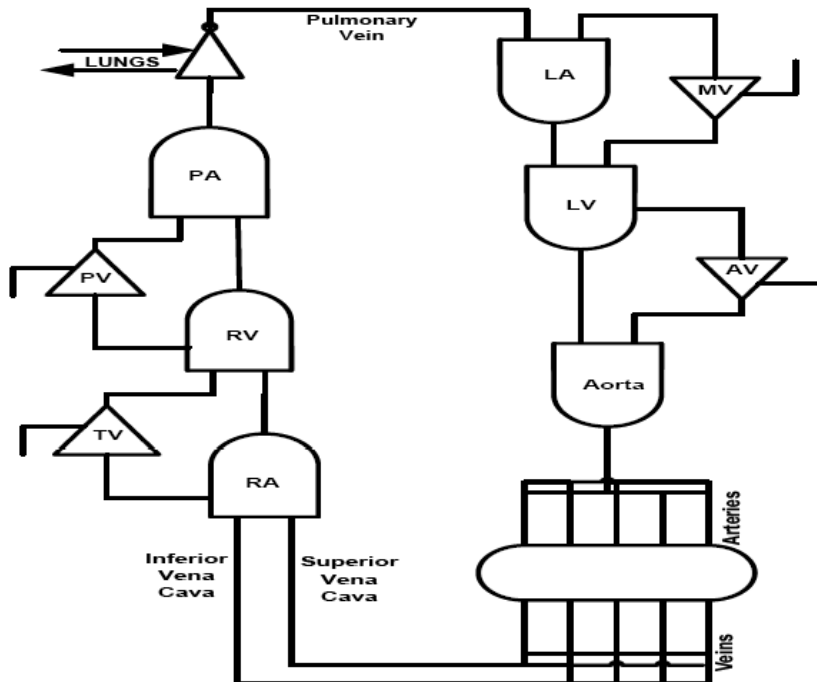


Figure 1: Logic Circuit Model of Heart Cycle

The flow of blood is in circular motion and output on the right side of the heart has an inverse input to the left side of the heart and vice versa, from the left side of the heart to the body and back to the heart. We can therefore represent the heart as follows: -

$$f(A+B) = f(\overline{A+B})$$

The next step is to individually look at the four parts of the heart in their entirety. Each Section of the heart, that is, both atriums and both ventricles have a preceding valve. We therefore define this in the following truth tables and expressions. Boolean variables are used as inputs and in logical function.

3.2 Deoxygenated Blood (Right Side)

Here we look at the right side of the heart, the right atrium, right ventricles and the valves (tricuspid and pulmonary valve).

- i) Deoxygenated blood flows from the superior and inferior vena cava to the Right Atrium (RA) when the RA is filled with blood, pressure is mounted and it contracts to open the Tricuspid Valve (TV) to allow forward movement of blood to the right ventricles. This is described in the truth table below:

Right Atrium (RA) and Tricuspid Valve (TV)

Deoxygenated blood In = A = {0, 1}

Right Atrium = RA = {0, 1}

Tricuspid Valve = TV = {0, 1}

$A \hat{=} f \{RA, TV\}$

Table 1: Shows state of Right Atrium (RA) and Tricuspid Valve (TV) in blood flow

RV	TV	A	Explanation
0	0	0	No activity
1	1	1	True State: Blood flows (forward movement)
0	1	0	low blood pressure there is no enough blood for valve to open;
1	0	0	Valve problem (Stenosis, Atresia) because this prevents blood from flowing out. It can also lead to RA leak or rupture.

From the truth table above, we derive the following equation to represent the blood flow in the Right Atrium.

$$A = f (RA \cdot TV)$$

- ii) Once deoxygenated blood flows into the RV the adjoining valves (TV and PV) need to work in co-ordination mainly to avoid back flows. Here the pressure that is mounted need to be high to push blood out of the heart and to the lungs.

Tricuspid Valve (TV), Right Ventricle (RV) and Pulmonary Valve (PV)

Deoxygenated blood out = B = {0, 1}

Tricuspid Valve = TV = {0, 1}

Right Ventricle = RV = {0, 1}

Pulmonary Valve = PV = {0, 1}

$B \hat{=} f \{TV, RV, PV\}$

Table 2: Shows state of Tricuspid Valve (TV), Right Ventricles (RV) and Pulmonary Valve (PV)

TV	RV	PV	B	Explanation
0	0	0	0	No activity
1	1	0	1	True State: Blood flowing into the Right Ventricle
1	1	1	0	TV problem causing back flow (Regurgitation)
0	1	1	1	True State: Blood flows (forward movement)
0	1	0	0	It happens but in a fraction of a second (not noticeable). If state remains PV has a defect (Stenosis, Atresia). It can cause RV to leak or rupture.
1	0	1	0	No activity

From the truth table above, we derive the following equation to represent the blood flow out of the Right Ventricles.

$$B = f(\overline{TV} + (RV \cdot PV))$$

3.3 Oxygenated Blood (Left Side)

Here we look at the left side of the heart, the left atrium, left ventricles and the valves (mitral and aortic valve).

- i) Oxygenated blood flows from the lungs to the Left Atrium, when the LA is filled with blood, pressure is mounted and it contracts to open the Mitral Valve (MV) through which blood flows to the left ventricles. This is described in the truth table below:

Left Atrium (LA) and Mitral Valve (MV)

$$\text{Oxygenated blood in} = \overline{A} = \{0, 1\}$$

$$\text{Left Atrium} = LA = \{0, 1\}$$

$$\text{Mitral Valve} = MV = \{0, 1\}$$

$$\overline{A} \triangleq f\{LA, MV\}$$

Table 3: Shows state of Light Atrium (LA) and Mitral Valve (MV) in blood flow Figure

LA	MV	\overline{A}	Explanation
0	0	0	No activity
1	1	1	True State: Blood flows (forward movement)
0	1	0	low blood pressure there is no enough blood for valve to open;
1	0	0	Valve problem (Stenosis, Atresia) because this prevents blood from flowing out. It can also lead to LA leak or rupture.

From the truth table above, we derive the following equation to represent the blood flow in the Left Atrium.

$$\overline{A} = f(LA \cdot MV)$$

ii) Here, we discuss how the blood flows from Mitral Valve to the Left Ventricles through the Aortic Valve and the rest of the body. Once oxygenated blood flows into the LV the adjoining valves need to work in co-ordination to maintain a forward movement, hence avoid any possibilities of back flows.

Mitral Valve (MV), Left Ventricle (LV) and Aortic Valve (AV)

$$\text{Oxygenated blood out} = \bar{B} = \{0, 1\}$$

$$\text{Mitral Valve} = MV = \{0, 1\}$$

$$\text{Left Ventricle} = LV = \{0, 1\}$$

$$\text{Aortic Valve} = AV = \{0, 1\}$$

$$\bar{B} \triangleq f\{MV, LV, AV\}$$

Table 4: Shows state of Mitral Valve (MV), Left Ventricles (LV) and Aortic Valve (AV)

MV	LV	AV	\bar{B}	Explanation
0	0	0	0	No activity
1	1	0	1	True State: Blood flowing into the Ventricle
1	1	1	0	MV problem causing back flow (Regurgitation)
0	1	1	1	True State: Blood flows (forward movement)
0	1	0	0	It happens but in a fraction of a second (not noticeable). If state remains AV has a defect (Stenosis, Atresia). It can cause LV to leak or rupture.
1	0	1	0	No activity

From the truth table above, we derive the following equation to represent the blood flow out of the Left Ventricles.

$$\bar{B} = f(\overline{MV} + (LV \cdot AV))$$

In conclusion, we combine all the four equations derived from the above four truth tables:

$$\text{Deoxygenated blood flow} \triangleq f(A + B)$$

Where,

$$A = f(RA \cdot TV), \text{ And}$$

$$B = f(\overline{TV} + (RV \cdot PV))$$

$$\text{Oxygenated blood flow} \triangleq f(\overline{A + B})$$

Where,

$$\bar{A} = f(LA \cdot MV), \text{ And}$$

$$\bar{B} = f(\overline{MV} + (LV \cdot AV))$$

Therefore, by equation we can say that the functioning of the Right side of the heart is inversely proportion to the functioning of the left side of the heart.

That is,

$$f(A+B) = f(\overline{A+B})$$

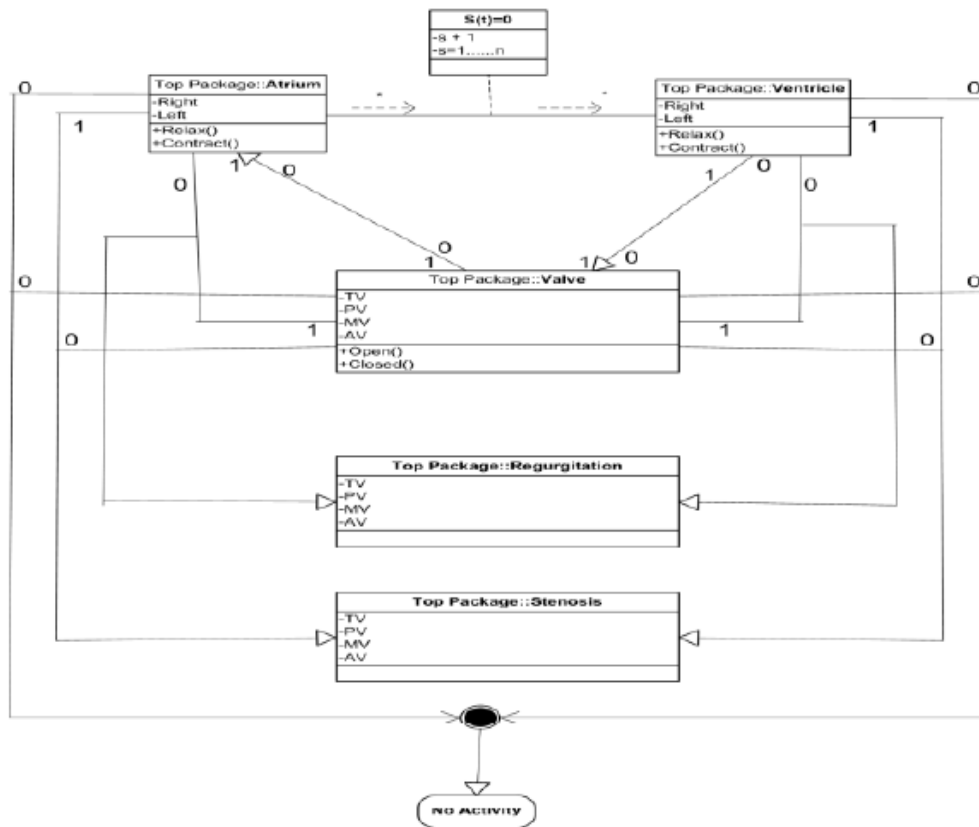


Fig 2: UML diagram for heart cycle model

The implementation of the model will assist in specific identification of the exact heart valve and/or chamber failure in relation to heart disease. In this paper we discuss each truth table and how the combination of binary variables can be used to classify various heart valve and chamber diseases.

Table 1 and Table 3: If RA/LA=0 and TV/MV=1, then this means that there is low blood pressure, for a valve to open, but in cases where the valve opens then we presume that TV/MV has a problem of flip flop (regurgitation) causing it to open whilst the RA/LA is still not full with blood hence no enough pressure due to mostly poor blood in circulations. Regurgitation will affect the next chamber of the heart, that is, the RV/LV.

If RA/LA=1 and TV/MV=0, means that there is enough blood in the right atrium to force TV opening, but as in this case the TV/MV is closed, this only means that there is valve failure referred to as stenosis, atresia or prolapse. This is where the TV/MV either fully or partially opens due to swelling of the valve opening or tissue flaps leading to minimal to no blood flow in cases where the tissue flaps close completely. This blood will then be forced to flow back into the right atrium a condition termed as regurgitation. The other possible scenario is RA/LA leak or rupture due to the high pressure in the chamber.

In case where RA/LA has no blood (0) and TV/MV is closed (0) could meant that there is no activity going on in that chamber of the heart, like blood flow which can imply death. The ideal state is where the

RA/LA=1 and TV/MV=1, this means that their enough blood pressure to force the TV/MV to open, thus forward movement of blood.

In Table 2 and Table 4: Here two valves intersect and the ventricles. For performance of the heart the two adjacent valves both in the right and left ventricles need to work in co-ordination. To maintain a forward movement of blood cycle TV/MV=0, and PV/AV=1. This means that the

TV/MV is closed allowing high pressure in the RV/LV to force PV/AV to open hence blood flow hence blood flow to the lungs and the rest of the body, irrespectively. The state where TV/MV=0, RV/LV=1, and PV/AV=0 should happen in a fraction of a second during the heart cycle, but if the state is prolonged, it will means the PV/AV is defective leading to heart diseases namely stenosis, Atresia or prolapsed where valve flaps are fused together or swollen making it difficult for blood to flow through. The high pressure in the RV/LV might lead to RV/LV leaks or rupture.

In cases where TV/MV=1, RV/LV=1 and PV/AV=1 means that the TV/MV flaps are faulty leading to a flip flop effect hence causing blood to flow backwards a condition termed as Regurgitation. Where TV/MV=1, RV/LV=1 and PV/AV=0 means that blood is flowing into the RV/LV and the PV/AV is closed to allow for this event. This is a true state of blood flowing into the RV/LV. Where TV/MV=0, RV/LV=0 and PV/AN=1, this means that the intersecting valves in the ventricles are open while there is no blood flowing in or out of the heart, this therefore means there is no activity of the heart and thus we can imply death. Where TV/MV=0, RV/LV=0 and PV/AV=0, means that the intersecting valves at the ventricles are closed and RV/LV contains no blood, with this we can also imply death since there is no activity in this chambers.

4 Mathematical Analyses

Using the global states of the heart cycle, the current state and input will determine the current output of the next state and likewise the behavior of the next state. This therefore means that the heart cycle will be composed of a series of states' **S**, each state giving a new state **S+1**, for each state there will be a current input **X** and a current output **Q**, and this will be time **t** bound.

Due to its nature of contraction and relaxation, we will define the heart cycle states in form of **0s** and **1s**, depicting diastole and systole respectively. This by default is a **two-state time trajectory** due to its alternating dynamism that alternates among the two binary values: 0 = relaxation and 1= contraction.

Given, initial state $S(t_i)$, at initial time $t_i = 0$, Current input= X , at initial time $t_i = 0$. To give rise to the next state ($S + 1$), we combine the current state to the current input, $S(t_i), X(t_i) = \delta(S(t_i), X(t_i))$. Output trajectory will then be, $\lambda(S(t_i), X(t_i))$. Therefore, for every state trajectory, for time $t_i = 0..upto..n$, where $n \leq \infty$, we will define our output trajectory for time t_i where $i = n$, as $\lambda(S(t_n), X(t_n))$.

4.1 State behavior (performance)

We can further explain our heart cycle by grouping it into similar functions, where, the left and right ventricle operate the same and at equal time and the left and right atrium operate the same and at equal times, that is contraction and relaxation, respectively and interchangeably. Here, we used the linear discrete time networks. For clarification, the atriums will be defined as **A**, and the ventricles as **V**, and illustrated by use of a matrix.

If, $V = 1$, then $A = 0$ at state $S(t_i)$, for time t_i where $0 \leq i \leq n$

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} A & V \\ -V & -A \end{bmatrix}$$

The -ve sign means that one side is inverse of the other in terms of input (oxygenated vs. deoxygenated).

Therefore,

$$[Current\ state] \cdot [current\ input] = [Next\ state] = [Current\ output]$$

$$\begin{bmatrix} 0 & V \\ -V & 0 \end{bmatrix} \cdot \begin{bmatrix} X_a \\ X_b \end{bmatrix} = \begin{bmatrix} \delta(V(t_i), X_a(t_i)) \\ \delta(-V(t_i), X_b(t_i)) \end{bmatrix} = \begin{bmatrix} \lambda(V(t_i), X_a(t_i)) \\ \lambda(-V(t_i), X_b(t_i)) \end{bmatrix}$$

As the cycle continues the Next state becomes the current state and so on. The next state is thus obtained by iteratively multiplying the current state with successive states. To obtain the next state above the current state was multiplied by $[1,1]$.

That is; $\begin{bmatrix} 0 & V \\ -V & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} V \\ -V \end{bmatrix}$

So, the consecutive state after $[V, -V]$, will be

$$\begin{bmatrix} 0 & V \\ -V & 0 \end{bmatrix} \cdot \begin{bmatrix} V \\ -V \end{bmatrix} = \begin{bmatrix} -V^2 \\ -V^2 \end{bmatrix} \dots \dots \begin{bmatrix} 0 & V \\ -V & 0 \end{bmatrix} \cdot \begin{bmatrix} -V^2 \\ -V^2 \end{bmatrix} = \begin{bmatrix} -V^3 \\ V^3 \end{bmatrix} \dots \dots \dots$$

Hence, the state trajectory will be as follows

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} V \\ -V \end{bmatrix}, \begin{bmatrix} -V^2 \\ -V^2 \end{bmatrix}, \begin{bmatrix} -V^3 \\ V^3 \end{bmatrix}, \begin{bmatrix} V^4 \\ V^4 \end{bmatrix}, \begin{bmatrix} V^5 \\ -V^5 \end{bmatrix}, \begin{bmatrix} -V^6 \\ -V^6 \end{bmatrix} \dots \dots$$

This will go on for infinite number of steps $\begin{bmatrix} V^n \\ V^n \end{bmatrix}$ Where, $n \leq \infty$.

Likewise, If $A = 1$, then $V = 0$ at state $S(t_i)$, for time t_i where $0 \leq i \leq n$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} A & V \\ -V & -A \end{bmatrix}$$

As stated above the -ve sign means that one side is inverse of the other in terms of input (oxygenated vs. deoxygenated). Therefore,

$$[Current\ state] \cdot [current\ input] = [Next\ state] = [Current\ output]$$

$$\begin{bmatrix} A & 0 \\ 0 & -A \end{bmatrix} \cdot \begin{bmatrix} X_a \\ X_b \end{bmatrix} = \begin{bmatrix} \delta(A(t_i), X_a(t_i)) \\ \delta(-A(t_i), X_b(t_i)) \end{bmatrix} = \begin{bmatrix} \lambda(A(t_i), X_a(t_i)) \\ \lambda(-A(t_i), X_b(t_i)) \end{bmatrix}$$

Next State; $\begin{bmatrix} A & 0 \\ 0 & -A \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} A \\ -A \end{bmatrix}$

So, the consecutive state after $[A, -A]$, will be

$$\begin{bmatrix} A & 0 \\ 0 & -A \end{bmatrix} \cdot \begin{bmatrix} A \\ -A \end{bmatrix} = \begin{bmatrix} A^2 \\ A^2 \end{bmatrix} \dots \dots \begin{bmatrix} A & 0 \\ 0 & -A \end{bmatrix} \cdot \begin{bmatrix} A^2 \\ A^2 \end{bmatrix} = \begin{bmatrix} A^3 \\ -A^3 \end{bmatrix} \dots \dots \dots$$

State trajectory will be as follows

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} A \\ -A \end{bmatrix}, \begin{bmatrix} A^2 \\ A^2 \end{bmatrix}, \begin{bmatrix} A^3 \\ -A^3 \end{bmatrix}, \begin{bmatrix} A^4 \\ A^4 \end{bmatrix}, \begin{bmatrix} A^5 \\ -A^5 \end{bmatrix}, \begin{bmatrix} A^6 \\ A^6 \end{bmatrix} \dots \dots$$

This will go on for infinite number of steps $\begin{bmatrix} A^n \\ A^n \end{bmatrix}$ Where, $n \leq \infty$, until such time that one of the heart valves or chamber experience a disease and the heart behavior changes from the norm.

We combine the state trajectory for both the atrium and ventricles as follows:

	$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$		$\begin{bmatrix} V \\ -V \end{bmatrix}$		$\begin{bmatrix} -V^2 \\ -V^2 \end{bmatrix}$		$\begin{bmatrix} -V^3 \\ V^3 \end{bmatrix}$		$\begin{bmatrix} V^4 \\ V^4 \end{bmatrix}$		$\begin{bmatrix} V^5 \\ -V^5 \end{bmatrix}$		$\begin{bmatrix} -V^6 \\ -V^6 \end{bmatrix}$		
$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$		$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$		$\begin{bmatrix} A \\ -A \end{bmatrix}$		$\begin{bmatrix} A^2 \\ A^2 \end{bmatrix}$		$\begin{bmatrix} A^3 \\ -A^3 \end{bmatrix}$		$\begin{bmatrix} A^4 \\ A^4 \end{bmatrix}$		$\begin{bmatrix} A^5 \\ -A^5 \end{bmatrix}$		$\begin{bmatrix} A^6 \\ A^6 \end{bmatrix}$	
	$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$		$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$		$\begin{bmatrix} V \\ -V \end{bmatrix}$		$\begin{bmatrix} -V^2 \\ -V^2 \end{bmatrix}$		$\begin{bmatrix} -V^3 \\ V^3 \end{bmatrix}$		$\begin{bmatrix} V^4 \\ V^4 \end{bmatrix}$		$\begin{bmatrix} V^5 \\ -V^5 \end{bmatrix}$		$\begin{bmatrix} -V^6 \\ -V^6 \end{bmatrix}$
$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$		$\begin{bmatrix} A \\ -A \end{bmatrix}$		$\begin{bmatrix} A^2 \\ A^2 \end{bmatrix}$		$\begin{bmatrix} A^3 \\ -A^3 \end{bmatrix}$		$\begin{bmatrix} A^4 \\ A^4 \end{bmatrix}$		$\begin{bmatrix} A^5 \\ -A^5 \end{bmatrix}$		$\begin{bmatrix} A^6 \\ A^6 \end{bmatrix}$			

0.83 seconds = 1 beat

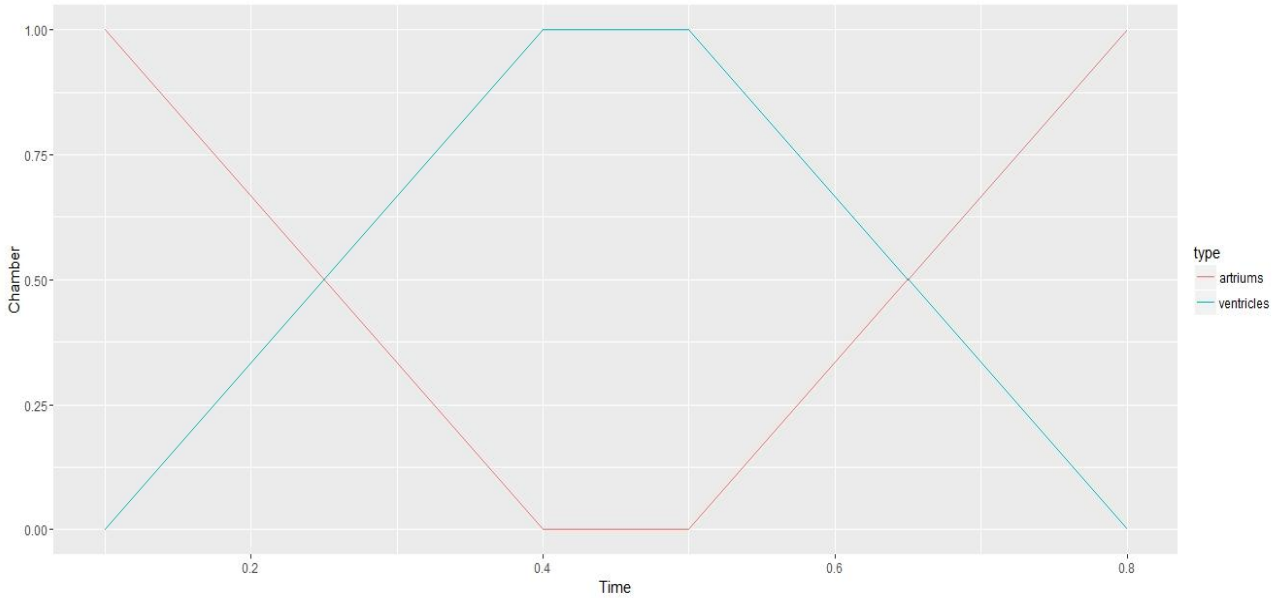


Fig 3: State trajectory for atriums and Ventricles

4.2 Output Behavior (blood flow)

In reference to the state trajectory, we will look at the output behavior in terms of the blood flow in the heart cycle. Similar to the operational behavior noted in analysis of the behavioral performance of the states above we will use differential equations to try and map the infinite series of the blood outflow within the finite number of states of the heart cycle.

To do this we will use vector quantities because of direction of the flow.

Our representation is as follows:

$$\text{State vector} = \vec{s}(t_i) , \text{Input vector} = \vec{x}(t_i) , \text{Output vector} = \vec{y}(t_i)$$

$$\text{State matrix} = A , \text{Input matrix} = B , \text{Output matrix} = Out$$

$$\text{Direct transition matrix} = D$$

$$\dot{\vec{s}}(t_i) = A(\vec{s}(t_i)) + B(\vec{x}(t_i)) \dots \text{First order differential equation or state equation}$$

$$\vec{y}(t_i) = Out(\vec{s}(t_i)) + D(\vec{x}(t_i)) \dots \text{output equation}$$

Our phase variables will be defined as:

$$S_1 = y, S_2 = \dot{y}, S_3 = \ddot{y}, S_4 = \dddot{y}, \dots, S_n = \frac{d^{n-1}y}{dt^{n-1}}$$

Similarly, our state variables will be as follows:

$$\dot{S}_1 = S_2, \dot{S}_2 = S_3, \dot{S}_3 = S_4, \dots, \dot{S}_{n-1} = S_n$$

We will simulate the first three states as follows by integrating the corresponding state equation.

$$\dot{S}_1 = S_2 \dots \dots \dots \dot{S}_1 = \int S_2 dt$$

$$\dot{S}_2 = S_3 \dots \dots \dots \dot{S}_2 = \int S_3 dt$$

$$\dot{S}_3 = S_4 \dots \dots \dots \dot{S}_3 = \int S_4 dt$$

Therefore, since it is a complete deterministic two-state trajectory, has timing references, and the pressure alternates for each chamber, we will therefore use a sine wave for precision and dynamic modeling and simulation. From our state variables above, we derive the following n^{th} order differential equation.

Where $x(t)$ represent state input and $y(t)$ represent state output.

$$x(t) = a_0 y + a_1 \frac{dy}{dt} + \dots \dots \dots + a_{n-1} \frac{d^{n-1}y}{dt^{n-1}} + \frac{d^n y}{dt^n}$$

To derive our sine wave, we will use the Fourier series to express our $x(t)$ as infinite series of sine and cosine functions.

$$\begin{aligned} x(t) &= a_0 + \sum_{n=1}^{\infty} (a_n \sin nt + b_n \cos nt) \\ &= a_0 + a_1 \sin t + a_2 \sin 2t + a_3 \sin 3t + \dots \\ &+ b_1 \cos t + b_2 \cos 2t + b_3 \cos 3t + \dots \end{aligned}$$

We will assume that the input function $x(t)$ is a sum and is a continuous function. From the above state integrals, at interval $(-\pi \leq t \leq \pi)$ we obtain the following equation.

$$\begin{aligned} \int_{-\pi}^{\pi} x(t) dt &= \int_{-\pi}^{\pi} a_0 dt + \int_{-\pi}^{\pi} \sum_{n=1}^{\infty} (a_n \sin nt + b_n \cos nt) \\ &= 2\pi a_0 + \sum_{n=1}^{\infty} a_n \int_{-\pi}^{\pi} \sin nt dt + \sum_{n=1}^{\infty} b_n \int_{-\pi}^{\pi} \cos nt dt \end{aligned}$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} x(t) dt$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} x(t) \sin nt dt \quad n = 1, 2, 3 \dots \dots \dots$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} x(t) \cos nt dt \quad n = 1, 2, 3 \dots \dots \dots$$

5. Conclusion

During our review on related works, we observed that recent published studies done on flow of blood and diseases in the heart chamber and valves used 3D hydrodynamics simulation while others were on numerical classifications and analysis. We used two approaches; binary variables that are theoretical and numerical, yet gives a clear insight into the complex dynamism of the operatives and interactions of all the four chambers and valves during a heart cycle; and mathematical analysis of the logic heart cycle model to be able to understand states behavior in relation to the atriums and ventricles and within discrete time.

A descriptive analysis of the heart cycle was done to enable careful mapping of the various states of the atriums, ventricles and the heart valves using binary representations. In our analysis of the binary representations of various probable states represented by the binary variables, we found that the blood in the heart travels in a cycle, and that output flow of blood on one side of the heart (for example, deoxygenated side) is the inverse and input of blood in another side of the heart (Oxygenated side). Also the performance of one chamber of the heart affects the performance of the other three (3) chambers; this is because the output of one chamber is the input to another chamber and forward movement of the blood is mainly dependent on the state of the valves.

Differential equations were used to map the infinite series of the blood outflow within the finite number of states of the heart cycle in an attempt of understanding the output behavior. We observed that the atriums and ventricles work interchangeably and distinctively together, respectively and that they inherit the overall behavior of relax (1) and contract (0) during their four transition states at an interval of 0.8 seconds representing 1 beat.

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