



Height Determination

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Abstract

The conventional definition of levelling rolling around the process of height difference determination. Thus, when the height of one point is known, then, heights of the other points can be calculated. Based on the height difference technique or method adopted, levelling can be classified into so many types such as ordinary, precise, trigonometric, barometric, tacheometric, GPS, etc.

To determine the reduced level or height of points, it is necessary to define a datum upon which heights of points are referred. The most important reference datums are the Geoid that can be represented by the mean sea level and the Ellipsoid which defines the mathematical surface of the earth.

The mean sea level that physically represents the base of orthometric heights is rising due to climate change. This rise is caused primarily by global warming which added water from melting land-based ice sheets and glaciers and the expansion of seawater as it warms. This future continuous rise leads to changes in ground reduced levels based on the mean sea level.

This research work tried to discuss methods of height determination that can be in direct contact with objects or at a distance. In addition to the reference datums adopted which may often either be geoid or ellipsoid.

The work concluded with that, geodetic heights based on ellipsoid as a reference datum could recently be easily determined through GPS and it may be transferred to orthometric heights since accurate measurement for geoidal model is available. Direct methods for height determination are suitable for a limited number of points where indirect ones could be used to collect data for large areas. Where the accuracy of the latter depends largely on the resolution and the number of ground control points used.

Since the mean sea level is continuing to change; it is not coinciding with the geoid where the world geodetic datum (WGS84) ellipsoid is now the dominant geodetic model.

Digital level is now replacing optical one where trigonometric levelling is now directly carried out with total station.

Keywords: *Barometric heighting, Ellipsoidal height, Geodetic height, Global Positioning System (GPS), Levelling, Orthometric height, Photogrammetry, Remote sensing, Side Airborne Looking Radar (SALR), Triangulation.*

Introduction

Leveling is the general term applied to any process by which heights / elevations of points or differences in heights / elevation are determined. It is a vital operation in producing the necessary data for mapping, engineering design and construction. Differences in heights can conventionally be determined by applying geometric levelling - or levelling in brief - which can be ordinary or precise using optical instruments or digital ones. Trigonometric and barometric leveling can also be used in addition to other aerial and satellite methods. The method to be selected depends primarily on the area to be covered, accuracy required and the nature of the terrain over which the levelling is to be carried out.

In land surveying, geometric leveling can produce better accuracy compared with the other techniques. Although trigonometric leveling produces a somewhat lower accuracy, it is

still suitable for many projects such as establishing vertical control for topographic mapping or for lower order construction setting-out. It is particularly convenient in hilly or mountainous terrain where large differences in elevations are encountered.

Barometric levelling can directly determine height of points with accuracy sufficient to multiple applications in which high accuracy does not require.

The Global Positioning System (GPS) can be used for control surveys. It is directly defining ellipsoidal heights. Therefore, it is necessary to consider geoidal undulations in the study area. Recently GPS leveling approach arise by the combination of the GPS derived ellipsoidal heights and the geoid information in order to determine orthometric heights ^[1].

Aerial photogrammetry and satellite imagery can also be another source of levelling data applying visible or invisible wavelengths that yields lidar and radar methods of height difference determination.

Orthometric heights

A reference datum is the surface to which the elevations of points are referred. It describes the height of the point above or below it, so it is necessary to calculate heights. The historical and dominant datum that used to define heights is the Geoid. The geoid is an equipotential gravitational surface, which is everywhere perpendicular to the direction of gravity. It can be physically represented by the Mean Sea Level (M.S.L). Heights reduced based on the geoid as a reference datum known as Orthometric heights.

It must emphasize that a geoidal surface differs from mean sea level. The mean sea level may be higher or lower than a geoidal surface because ocean currents, temperature, salinity, and wind variations can cause persistent high or low areas in the ocean. These differences are measurable, in places over a meter (3 feet), perhaps small on a global scale, but large in local or regional analysis [11]. It historically referenced heights to mean sea level, this is no longer true for most spatial data analyses.

Specialists noted that the Global Mean Sea Level (GMSL) is changing during both old tide gauge measurement and modern satellite altimetry measurement. It has changed since 1993 up to 2020 by about ten centimeters (96.7 ± 4 mm) and with approximately 3.34 mm rate of change [12].

Table 1: Change of the global mean sea level

Period	Rate of GMSL change (mm/year)	Measuring method
1900 - 2018	1.56	Tide Gauges
1993 - 2018	3.35	Tide Gauges
1993 - 2020	3.34	Satellite altimetry

Although this change is critical for all the planet, it is directly affecting orthometric height measurements based on the mean sea level.

The geoid can be calculated from terrestrial gravimetric observations. It is a measured and interpolated surface, and not a mathematically defined surface. The geoid's surface is measured using several methods, initially by a combination of plumb bob - a weight suspended by a string that indicates the direction of gravity - and horizontal and vertical distance measurements, and later with various types of gravimeters devices that measure the gravitational force.

Satellite-based measurements in the late 20th century substantially improved the global coverage, quality, and density of geoidal height measurements. The Grace experiment, initiated with the launch of twin satellites in 2002, is an example of such improvements. Distances between a pair of satellites are constantly measured as they orbit the Earth. The satellites are pulled closer or drift farther from the Earth due to variation in the gravity field. Because the orbital path changes slightly each day, it eventually has nearly complete earth coverage of the strength of gravity, and hence the location of the reference gravitational surface [11].

Ellipsoidal heights

On the other hand, the alternative reference datum that height of points can be referred is Ellipsoid. The ellipsoid is a mathematical surface obtained by revolving an ellipse about the Earth's polar axis. Heights reduced based on this datum is known as Ellipsoidal or Geodetic heights.

The ellipsoid that represents the earth surface needs to be defined by the semi major axis (a) and the semi minor axis (b) as shown in the figure (1) Below.

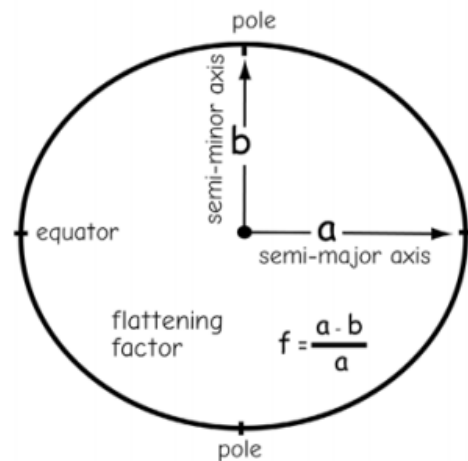


Fig. 1: An ellipsoidal model of the earth's shape.

From the nineteenth century, many countries had started national Astro-geodetic surveying, aimed at determining the size of the Earth ellipsoid, and, more importantly, providing accurate geometric positions of numerous surface points for national topographic mapping.

In physical geodesy the calculation of the distance between surface points and the reference ellipsoid (i.e., geodetic height) can be calculated from terrestrial gravimetric observations.

With the maturation of GPS technology, since the 1980s GPS surveying has already become the chief method of geodetic measurements. GPS positioning techniques enable high-accuracy measurement of the geocentric coordinates of any surface point in the world geodetic coordinate system based on the precisely measured elements of satellite orbit. The use of the satellite radar altimetry technique to measure geoid undulation of the seawater has also yielded good results.

Regional (relative) geodetic measurement in the local reference coordinate system has been developed into global (absolute) geodetic measurement in the unified geocentric coordinate system.

Since some levelling techniques based on geoid as a reference datum and others based on ellipsoid, it is necessary to relate one system to another. For consistency between the two types of data it is necessary to consider the geoid ellipsoid separation (N) as illustrated in the figure below.

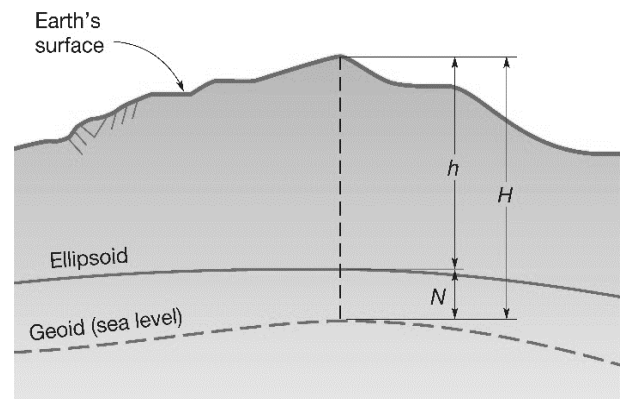


Fig. 2: Relationships between orthometric height and geoidal height.

Then the orthometric height (H) can be calculated as:

$$H = h + N$$

Where H is elevation above the geoid (orthometric height), h the geodetic height.

Previous studies showed that the accuracy in levelling for GPS data for height differences is better than the accuracy of orthometric height determination. Therefore, GPS can be used in many engineering applications when variation in height is the main target. Moreover, GPS data can be improved using gravitational model [1].

Direct levelling techniques

Classification of levelling can be suggested according to whether the height of point is directly determined through physical contact or indirectly at a distance. Thus, direct levelling technique can include ground surveying methods such as geometric, trigonometric barometric etc., in addition to GPS. On the other hand, indirect techniques can include aerial and satellite methods such as photogrammetry and aerial or satellite, lidar and radar techniques.

The most common method in direct levelling technique is the geometric levelling that can be ordinary or precise. It is based on using a special instrument known as a level; used to create a horizontal line of sight. Difference in elevation (ΔH) is the difference between the two readings (R_A) and (R_B) taken at a graduated staff held vertically over each of the two points whose height difference is to be determined. Then,

$$\Delta H = R_A - R_B$$

For long distances the effect of the curvature of the earth and atmospheric refraction can be considered as shown in the figure below.

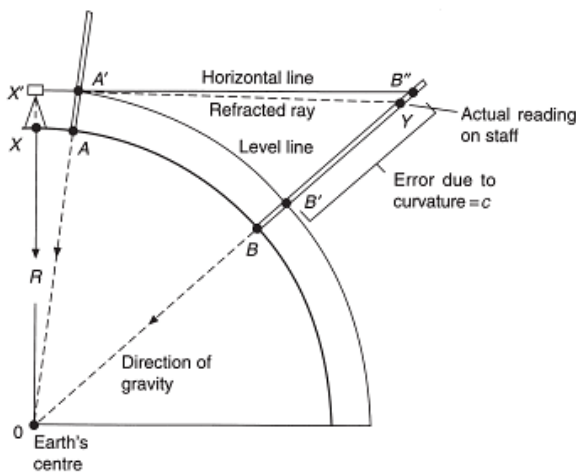


Fig. 3: The effect of earth curvature and atmospheric refraction on levelling.

From the figure it can be proved that the combined effect of the earth curvature (c) and atmospheric refraction (r) can be calculated as $(c - r)$ which is approximately equal to $0.0673D^2$, i.e. $(c - r) = 0.0673D^2$. Where D is the distance between the instrument and observed point (length of sights) in km.

$$\Delta H = R_A - R_B + 0.0673D^2$$

So, in tertiary levelling, where the length of sight is generally 25–30 m, the effect may be ignored [8].

Nowadays, a motorized geometric levelling application has been done by establishing survey hardware on the land vehicle, thus successful results have been obtained.

Where difficult terrain, such as mountainous areas, precludes the use of conventional levelling, trigonometrical levelling is used. It may also be used where the height difference is large, but the horizontal distance is short such as heighten up a cliff or a tall building [8]. The vertical angle (α) and the slope distance (D) between the two points concerned are measured. Slope distance is measured using Electromagnetic Distance Measurers (EDM) and the vertical (or zenith) angle using theodolite. Then,

$$\Delta H = h_i + D \times \sin \alpha - h_s$$

Where, h_i is the height of the instrument and h_s is the height of the signal set on the other point.

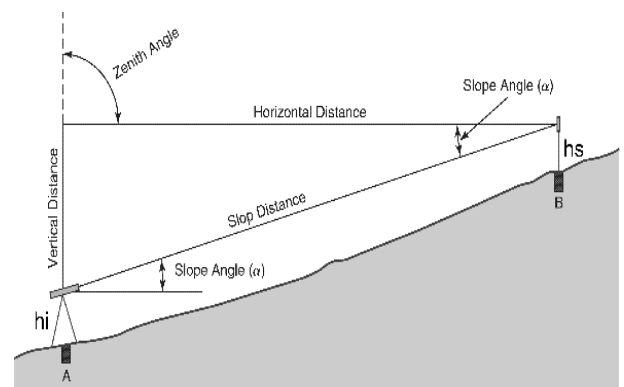


Fig. 4: trigonometric levelling.

When long distances are to be measured, it will be necessary to consider both the earth's curvature and atmospheric refraction as illustrated in the figure hereunder.

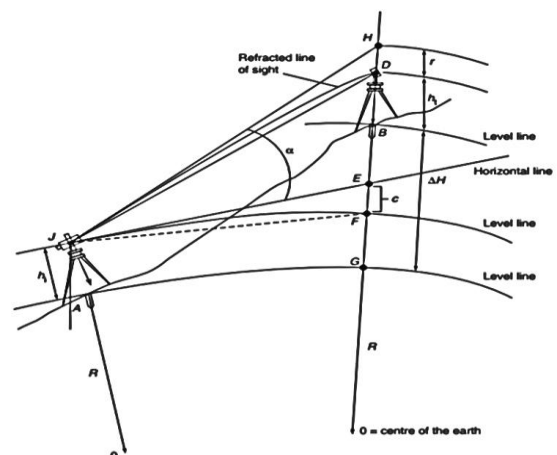


Fig. 5: Trigonometric levelling in long lines.

The difference in level ΔH can be calculated as:

$$\Delta H = h_i + D \times \sin \alpha - h_s + 0.0673D^2$$

Now a days the total station is used in trigonometric levelling instead of theodolite and electromagnetic distance measurement.

Modern Global Navigation Satellite Systems (GNSS), such as a Global Positioning System (GPS), can locate the position of points. These systems can be categorized as a direct method of levelling techniques since the instrument is located directly at the point.

GPS Structured from three components. The first component is the space segment that consists originally of 24 satellites operating in 6 orbital planes spaced at 60° interval around the equator. The satellites travel in near-circular orbits that have a mean altitude of 20,200km above the earth and an orbital period of 12 sidereal hours. The second component is the control segment which consists of five monitoring ground stations at which the signals from the satellites are monitored and their orbits tracked. The tracking information is relayed to the master control station in Colorado Springs. The master control station uses this data to make precise near future predictions of the satellite orbits, and their clock correction parameters.

The third component of the GPS segments is the user segment. It consists of two categories of receivers that are classified by their access to two services that the system provides; The Standard Positioning Service (SPS) provided at no cost to the user with lower accuracy and the Precise Positioning Service (PPS) that is only available to receivers having valid cryptographic keys with higher accuracy [1].

For the receiver to determine the ground position of the station they occupy, it was necessary to devise a system for accurate measurement of signal travel time from satellite to receiver. This was accomplished by modulating the carrier with Pseudo Random noise (PRN) code.

GPS receiver compute distances to four satellites and fixes a position by trilateration by applying either the code range or carrier phase-shift measurements as shown in figure (6) below.

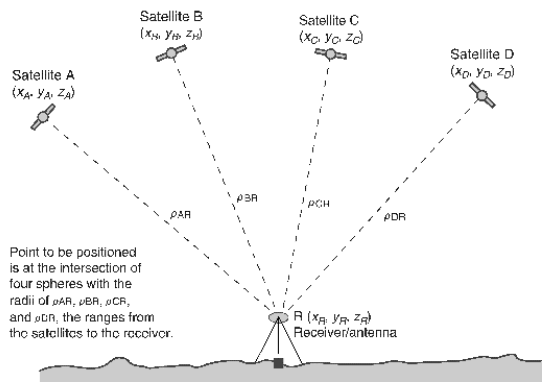


Fig. 6: Position fixing by GPS.

The coordinates of GPS single point positioning and the baseline vector in relative positioning solution belong to the WGS84 geodetic coordinate system on which the GPS satellite ephemeris is based. However, practical measurement results often belong to a national or local coordinate system. In real applications, one needs to solve the transformation parameters to transform coordinates. Ellipsoid to geoid to transformation should be considered to determine elevation of points.

Levelling can also be carried out using barometric heighting. It is a type of direct contact leveling in which differences of height are determined from differences of atmospheric pressure observed with barometers or altimeters. The main principle of barometric leveling is the difference between the elevation of two points is proportional to the difference between the atmospheric pressure of the points. Because levelling takes time, and the atmospheric pressure does not remain constant throughout the day. Therefore, they give only a rough estimation.

Indirect contact levelling techniques

These methods include all, at a distance measuring technique such as Photogrammetry, Lidar, or Side looking radar including both aerial and satellite platforms.

Photogrammetry is the art science and technology of extracting useful quantitative and qualitative information about physical and man-made objects by measurements and observations on photos and/or images of these objects. Although of these photographic constraints, some of digital photogrammetric data acquisition has an ability to collect images with wave lengths beyond the range of photographic region.

In aerial photogrammetric techniques the vertical photographs are normally exposed in such a way that the area covered by each successive photographs along a flight strip duplicate or overlaps part of the coverage of the previous photo. Then the three coordinates of any point can be obtained by measuring the two-dimensional photo coordinates (x₁, y₁) and (x₂, y₂) of the two successive photographs in which they appear, to determine the ground three coordinates (X, Y, Z) through optical, analytical, or digital techniques. Ground control points are required to complete this process. Also, atmospheric refraction distortion and earth curvature are considered.

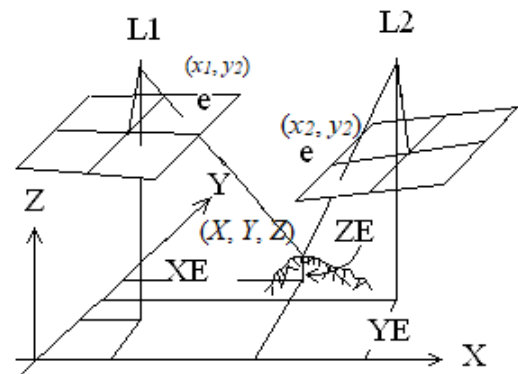


Fig. 7: Principle of photogrammetry.

In digital photogrammetry, digital photographs are stored and processed on a computer. Digital images can be scanned from photographs or can be directly captured by digital cameras. Many photogrammetric tasks can be highly automated in digital photogrammetry. The output products are in digital form, such as large scale topographical or thematic maps, Digital Elevation Models (DEMs), and digital orthophotos saved on computer storage media. Therefore, they can be easily managed, and used.

Recently GPS is used to determine the photogrammetric ground coordinates or to determine the exposure station coordinates and orientation data.

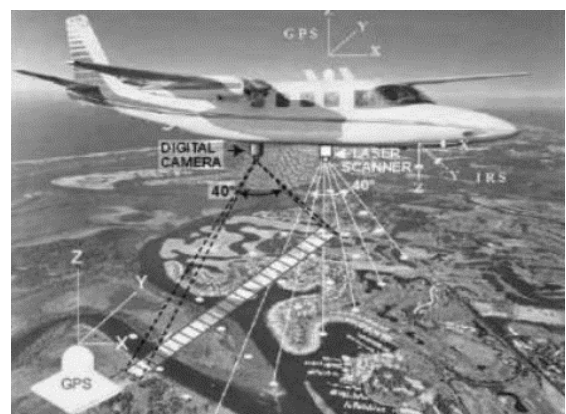


Fig. 8: Photogrammetry and GPS.

Indirect levelling methods also include Airborne LIDAR. It is an active sensory system that uses light to measure distances. This device can rapidly measure distances between the sensor on the airborne platform and points on the ground to collect and generate accurate elevation data.

A LIDAR device mounted in an airborne platform emits fast pulses from a focused infrared laser which are beamed toward the ground across the flight path by a scanning mirror. Upon capture by a receiver unit, the reflectance from the ground, and a time interval meter which measures the elapsed time between the transmitted and received signal. From this information, the distance separating the ground and airborne platform is determined. While in flight, the system gathers information on a massive base of scattered ground points and stores them in digital format. An interfaced Inertial Measurement Unit (IMU) records the pitch, roll, and heading of the platform. A kinematic airborne GPS system locks on to at least four navigation satellites and registers the spatial position of the aircraft. Additionally, many systems include a digital camera to capture photographic imagery of the terrain that is being scanned. Some systems have incorporated a video camera for reviewing areas collected.

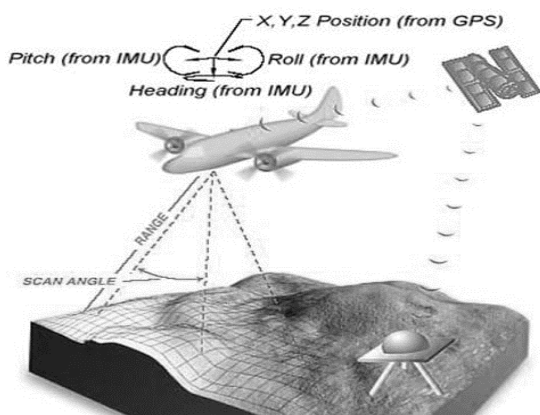


Fig. 9: Airborne Lidar system.

A typical technique is adopted in Satellite Laser Ranging (SLR) system. It is a physical distance-measuring method, using the laser as its light source and the time of flight of the optical pulse for measurements. It is one of the techniques that can carry out precise measurements. Many satellites such as meteorological satellites, Earth resources satellites, and oceanic satellites have all been equipped with laser reflectors to carry out more precise measurements. The SLR system consists of two main segments, a laser ranger on the ground and a laser satellite in space. The hardware devices of the ranger include laser, telescope, electro-optical head, pulse position measurement system, time and frequency system, servo system, and computer [9].

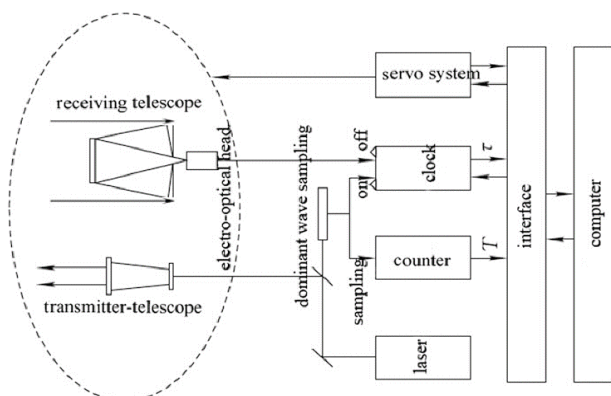


Fig. 10: Satellite Laser Ranging components.

Radar technology can also be used to collect topographic data utilizing airborne or satellites. In the Side Looking Airborne Radar (SLAR) system, the platform (aircraft or satellite) travels forward in the flight direction with the nadir directly beneath the platform. The microwave beam is transmitted obliquely at right angles to the direction of flight illuminating a swath. Range refers to the across-track dimension perpendicular to the flight direction, while azimuth refers to the along-track dimension parallel to the flight direction.

Swath width refers to the strip of the Earth's surface from which data are collected by a side-looking airborne radar. It is the width of the imaged scene in the range dimension. The longitudinal extent of the swath is defined by the motion of the aircraft with respect to the surface, whereas the swath width is measured perpendicularly to the longitudinal extent of the swath.

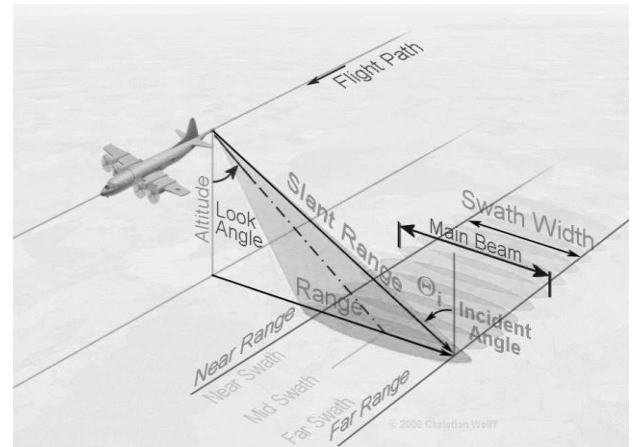


Fig. 11: Side looking airborne radar.

Previous studies such as [2,6,7] etc. found that indirect methods are suitable for collecting topographic data for large areas and the accuracy of aerial and satellite techniques depends largely on the resolution and number of ground control points used.

Conclusion

This work aimed to discuss levelling and height difference determination utilizing different methods. These methods vary in techniques, reference datum, accuracy, and applications. By referring to the review and discussion carried out above, it can be concluded with the following:

- To determine the height of a point it is necessary to define the reference datum. Although, a local datum can be used but geoid and ellipsoid are the main references.
- The mean sea level is continuing to change; therefore, it is suitable to use gravitational method to determine the geoid.
- The world geodetic datum (WGS84) ellipsoid is now the dominant geodetic reference datum used because of the spread use of GPS applications.
- Ellipsoidal heights obtained by GPS can immediately be transferred to orthometric heights.
- Resolution and the number of ground control points used are largely affecting the accuracy of indirect methods that are suitable for collecting data for large areas.
- Digital level is now replacing optical one where trigonometric levelling is now directly carried out with total station.

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